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Commentary

The Symmetry Principle

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Abstract: The symmetry principle is described in this paper. The full details are given in the book: J. Rosen, Symmetry in Science: An Introduction to the General Theory (Springer-Verlag, New York, 1955).

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1. Introduction

This is a very brief review of the derivation of the symmetry principle (also called Curie's symmetry principle, or the Curie principle) and some of its implications. For more detail see Rosen [1].

Pierre Curie stated (in translation) [2]: It is asymmetry ["dissymétrie" in the original] that creates a phenomenon. Paul Renaud generalized Curie's idea and stated (again in translation) [3]: If an ensemble of causes is invariant with respect to any transformation, the ensemble of their effects is invariant with respect to the same transformation. I have stated the symmetry principle as [1]: The symmetry group of the cause is a subgroup of the symmetry group of the effect. Or less precisely: The effect is at least as symmetric as the cause.

We will now see how the symmetry principle is derived from the existence of causal relations in nature. Then I will point out a number of the principle's implications, which can be considered to be principles in themselves. Finally, I will connect the symmetry principle with the second law of thermodynamics.

2. Causal relation

Consider a system that can be in any one of a set of states**.** The terms "system" and "state" are to be understood very generally. Consider a pair of subsystems of the system, A and B, such that the state of the system determines the state of each subsystem. The set of states of the system determines a correlation between states of one such subsystem and those of another and thus defines a mapping between the set of states of A and that of B. If the mapping is many-to-one (including one-to-one) from A's state set to B's, then A and B are in causal relation, with A the cause and B its effect. If the mapping is one-to-one, either subsystem can be considered the cause and the other its effect.

3. The equivalence principle

Recognized causal relations in nature are expressed as laws. Laws impose equivalence relations in the state sets of causes and of effects. (For a lengthy discussion of this point see Rosen [1].) It follows:

Equivalent states of a cause are mapped to (i.e., correlated with) equivalent states of its effect.

This is The Equivalence Principle. Also less precisely:

Equivalent causes are associated with equivalent effects.

4. The symmetry principle

Here we will go into somewhat more detail. Consider a system and a law governing it. Any state u of the system implies state u_c for some cause subsystem and state u_e for a corresponding effect subsystem. For the state set of the system define cause-equivalence, denoted $\equiv_c$, as follows: Two states of the system are cause-equivalent if and only if the states of the cause subsystem that they imply are equivalent. Symbolically:

$$u_c \equiv v_c \quad \Leftrightarrow \quad u \equiv_c v$$

for states u and v of the system. Similarly define effect-equivalence, denoted $\equiv_e$:

$$u_e \equiv v_e \quad \Leftrightarrow \quad u \equiv_e v.$$

The equivalence principle can be expressed symbolically:

$$u_c \equiv v_c \quad \Rightarrow \quad u_e \equiv v_e.$$

It follows that cause-equivalence in the system's state set implies effect-equivalence:

$$u \equiv_c v \quad \Rightarrow \quad u \equiv_e v.$$

Define the symmetry group of the cause as the symmetry group of the system for cause-equivalence in its state set. This is the group of all invertible automorphisms of the system's state set that preserve cause-equivalence. (In other words, it is the group of all permutations of the system's state set that keep states within their cause-equivalence classes.) Similarly, the symmetry group of the effect is defined as the symmetry group of the system for effect-equivalence. Since cause-equivalence implies effect-equivalence, it follows that every element of the symmetry group of the cause must necessarily also be an element of the symmetry group of the effect. (However, there might be elements of the latter that are not elements of the former.) This proves The Symmetry Principle:

The symmetry group of the cause is a subgroup of the symmetry group of the effect.

And less precisely:

The symmetry of the effect is at least that of the cause (and might be greater).

5. The equivalence principle for processes

The equivalence and symmetry principles can be applied to processes in isolated (more precisely, quasi-isolated) physical systems, whose evolution serves as the "system," a particular process as a state of the "system," the initial state as the "cause subsystem," and the final state (which is uniquely determined by the initial state) as its "effect subsystem." The laws of nature that describe such evolutions impose equivalence relations in sets of such initial states and final states. From the equivalence principle there follows The Equivalence Principle for Processes in isolated physical systems:

Equivalent initial states must *evolve into equivalent states*
(while inequivalent states may *evolve into equivalent states).*

6. The general symmetry evolution principle

In applying the reasoning of section 4 to processes in physical systems, we can use the more descriptive terms of initial- and final-equivalence for cause- and effect-equivalence, respectively, and "initial" symmetry group and "final" symmetry group for symmetry groups of the cause and of the effect. The result is The General Symmetry Evolution Principle:

The "initial" symmetry group is a subgroup of the "final" symmetry group.

This can also be stated as:

For an isolated physical system the degree of
symmetry cannot decrease as the system evolves,
but either remains constant or increases.

Note that here "degree of symmetry" has to do with the "initial" and "final" symmetry groups. It does *not* refer to any symmetry of the states of the physical system as the latter evolves.

When we consider the evolution of an isolated physical system, we normally consider the sequence of states it passes through and their symmetry and are usually not interested in the entire state set of the "system," in terms of whose automorphisms the "initial" and "final" symmetry groups are defined. Thus we find that the general symmetry evolution principle is rather useless in practice.

7. The special symmetry evolution principle

Let us now consider, instead, the symmetry group of a state of an evolving physical system. It is the group of all invertible automorphisms of the state set of the physical system that do nothing else but carry the state into equivalent states, where equivalence is imposed by the laws of evolution. The significance of equivalence is that equivalent states are indistinguishable by the laws of evolution [1]. Thus the symmetry group of a state is just the group of permutations of all members of the state's equivalence class in the state set. So the order of a state's symmetry group and the state's degree of symmetry are both directly related to the population of the state's equivalence class. As an example, macroscopic laws of evolution impose equivalence relations in the set of microstates, where the macrostates whose evolution the laws govern are the corresponding equivalence classes.

In order to be able to make a statement about the degrees of symmetry of the sequence of states through which an isolated system evolves, we need to make the assumption of nonconvergent evolution: Different states always evolve into different states. Then, by the equivalence principle for processes, the population of the equivalence class of the final state is at least equal to that of the initial state that evolved into the final state. Moreover, additional states, inequivalent to the initial state, may also evolve into members of the final state's equivalence class (convergence of equivalence classes), and these members will be distinct from those we just counted. Hence:

As an isolated system evolves,
the populations of the equivalence classes of the
sequence of states through which it passes cannot decrease,
but either remain constant or increase.

Equivalently:

The degree of symmetry of the state of an isolated system
cannot decrease during evolution,
but either remains constant or increases.

This is The Special Symmetry Evolution Principle.

8. Further implications

Pursuing the example in the preceding section, let us suppose that some macrostate M is a macrostate of stable equilibrium. Then many different macrostates will evolve into M. Since microstates are assumed not to converge during evolution, the population of the final equivalence class that is M is much larger than the population of the equivalence class of any microstate state that evolves into a final microstate belonging to M. The population of the equivalence class that is M in fact equals the sum of the populations of all equivalence classes whose member microstates evolve into a member microstate of M. So:

The degree of symmetry of a macrostate of stable equilibrium must be relatively high.

This is a general theorem. Although the symmetry involved in it is with respect to permutations of equivalent microstates, it can have macroscopic manifestations. For instance, in the case of an isolated gas (whose microstates indeed evolve nonconvergently) in a container, all macrostates of stable equilibrium are homogeneous (i.e., symmetric under all permutations of macroscopic subvolumes of the gas). This theorem is in accord with observation [4].

Under the correspondence

$$\text{degree of symmetry} \leftrightarrow \text{entropy}$$

the special symmetry evolution principle and the second law of thermodynamics are isomorphic. Both are concerned with the evolution of isolated systems; both state that a quantity, a function of macrostate, cannot decrease during evolution. So, following good science practice, we assume that a fundamental relation exists between entropy and degree of symmetry and look for a functional relation giving the value of one as a strictly monotonically ascending function of the other. Such a function is already known, however, from the statistical definition of entropy,

$$S = k \log W,$$

where W is the number of microstates corresponding to the macrostate for which the value of the entropy, S, is thus defined. (k denotes the Boltzmann constant.) As mentioned above, W can be used to measure the degree of symmetry of the macrostate, and the function $k \log W$ is indeed a strictly monotonically ascending function of W. See Sellerio [5].

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