Clifford quantum computer and the Mathieu groups

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Abstract. One learned from Gottesman-Knill theorem that the Clifford model of quantum computing [*Generalized Clifford groups and simulation of associated quantum circuits*, S. Clark, R. Jozsa and N. Linden, Quant Inf Comp 8, 106-26 (2008)] may be generated from a few quantum gates, the Hadamard, Phase and Controlled-Z gates, and efficiently simulated on a classical computer. We employ the group theoretical package GAP for simulating the two qubit Clifford group $\mathcal{C}_2$.

We already found that the symmetric group $S(6)$, aka the automorphism group of the generalized quadrangle $W(2)$, controls the geometry of the two-qubit Pauli graph [*On the Pauli graphs on N-qudits*, M. Planat and M. Saniga, Quant Inf Comp 8, 127-46(2008)]. Now we find that the *inner group* $\mathrm{Inn}(\mathcal{C}_2) = \mathcal{C}_2/\mathrm{Center}(\mathcal{C}_2)$ exactly contains two normal subgroups, one isomorphic to $\mathcal{Z}_2^{\times 4}$ (of order 16), and the second isomorphic to the parent $A'(6)$ (of order 5760) of the alternating group $A(6)$. The group $A'(6)$ stabilizes an *hexad* in the Steiner system $S(3,6,22)$ attached to the Mathieu group $M(22)$. Both groups $A(6)$ and $A'(6)$ have an *outer* automorphism group $\mathcal{Z}_2 \times \mathcal{Z}_2$, a feature we associate to two-qubit quantum entanglement.