Internal entropy as a measure of the amount of disorder of the subject-dependent environment

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Abstract: *Entropy* measures disorder of a system from the viewpoint of an external observer. However, such a measure cannot effectively describe and explain the degree of disorder of the environment an internal participant actually experiences. Such an aspect is important for understanding a fundamental principle of life: for survival, living systems (entities) need to manage the uncertainty of occurring events in order to maintain a particular favorable relationship with their surroundings. Events experienced by a subject, as a component of a system, depend on the actions of the subject. In other words, the actions produce a set of events occurring to the actor with a particular probability distribution. Such an action-dependent set of events occurring forms the environment of the actor. Then, the concept of disorder of the subject-dependent environment can be considered. In this paper, the probability of a subject-dependent event, called *internal probability*, and a measure of the disorder of the subject dependent environment (*internal entropy*) are introduced, using the cognizers-system model: system components are described as cognizers, acting each moment by cognizing other components of the system. The internal entropy may not necessarily increase with time; it may even decrease.

Keywords: adaptation; cognition; information; internal probability; internal entropy; uncertainty

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1 Introduction

One of the most important features of living systems is the ability of self-maintenance of organizational order against disturbing, uncertain environmental events. Living systems maintain a particular configuration among components, keeping themselves within a relatively small region of a much larger phase space, that is, holding themselves in low-entropy states. This ability appears to violate the second law of thermodynamics: disorder increases with time. However, it has been explained that living systems are open systems, which can maintain low-entropy states by taking a low-entropy form of energy from foods, and losing a high-entropy form of energy, heat; they are feeding on negative entropy [1]. Therefore, there is no contradiction to the second law.

However, while the above explanation is nothing more than stating that living systems do not violate the second law, it remains unclear how living systems can maintain their internal order while exchanging materials, and how they cope with their uncertain environments. Entropy indicates uncertainties about the system-states for the external observer, measuring the degree of uncertainty or disorder of the states. Specifically, an entropy value, which is given to a system's macro-state, refers to the degree of indetermination about its micro-states which the external observer cannot discriminate. An explanation is needed to illustrate how living systems can cope with uncertain environments, that is, a theory about the entropy of the environment for a subsystem, not of the entropy of the whole, closed system. To this problem, the ordinary entropy concept simply cannot be applied: such an environment is not independent of the subject, instead it is formed by mutual interactions between the subject and the environment.

Disorder of environments is a serious problem for living systems, such as cells and organisms, which may disturb their internal order, and therefore disrupt survival. Actions taken by living systems in their world affect types and probabilities of events occurring to them. If the ability of self-maintenance of organizational order of living systems against disturbing uncertainties in the surrounding environment is to be understood, a new theory is required.

In this paper, as an initial step towards a comprehensive theory for this problem, a mechanism by which subject-dependent probabilities of events, called *internal probabilities*, are determined and the relationships between the probabilities of the events that a subject experiences and the actions that the subject takes, is explicated. For this analysis, an agent-based, deterministic model of a system, called *cognizers system model*, developed by

Nakajima [2,4], is used to analyze relationships between the actions and the uncertainty of the environment that the actor experiences, and introduces a measure for this kind of disorder. The measure, called *internal entropy*, describes the degree of uncertainty about events occurring to an internal participant in the system or world, not to a universal observer located outside the world.

2 Theoretical scheme and concepts

Overview of theoretical scheme

Two different kinds of observer are possible in relation to the world: one located outside the world as an epistemological entity, the other located inside as a material entity participating in the world. As above, the focus is placed on the uncertainty of environments of living systems as internal participants; therefore, a world model, in which components of the world are subjects of observation, is needed.

Furthermore, because of our focus on uncertainty of events, a metaphysical aspect of world, whether it is deterministic or indeterministic must be defined. At least three meta-physical stances are possible when modeling a world concerning the origin of uncertainties. A world is

- deterministic, but observers lack perfect knowledge of it, and where uncertainties come from the limits of cognitive ability;
- itself, uncertain in its state change with time, i.e. an indeterministic world; or,
- indeterministic, and observers lack perfect knowledge.

In this paper, a deterministic world model is chosen: it may be better at explaining uncertainties without assuming an uncertain world in the first place— see Nakajima [3] for another reason. It should be noted that the present model should not be based on a classical, Laplacian view of uncertainty [5] where the world is deterministic, but observers lack perfect knowledge of it, and where uncertainties come from.

For this classical view, uncertainty about events arises from epistemological limitations of the external observer. However, uncertainties explicated are things that are experienced by material components of the world, not any external, epistemological entity. Therefore, a world model, in which material entities are described as subjects or agents, which senses the environments and acts accordingly, is required. In other words, motion is realized as sensation-and-action, i.e. *cognition*, by which the world evolves.

Cognition

Every material entity can be realized as a subject of action, an agent in the world. Action involves sensation; how an entity acts depends on the environment. This sensation-and-action is defined as *cognition* and determines the succeeding state of the cognizer depending on the current states of itself and the environment. Therefore, cognition includes any kind of change in state of material entity responding to the environmental state not limited to mental or neural processes of animals, including human. This can be extended to every material entity, including molecules, cells, organisms, and so on. Therefore, cognition generalizes the concept of motion, including not only changes in position but also in any kind of state-space. In this sense, every material entity is a cognizer.

Cognizers system

The cognizers-system model is a system composed of cognizers. Cognition is formalized as mapping from a current state to another in response to the current state of the environment. In the model, each cognizer takes a particular definite state at each point of time, which is mapped to another state in response to other cognizers inhabiting the system (world).

Consider a two-cognizers system. This system (world) is made up of only two cognizers, denoted as A and B, respectively. In this system, B is the environment for A, and vice versa. Suppose that A is in state ax, and B in bx, at a given time. ax is mapped to ay by cognizing B in bx, while bx is mapped to by by cognizing A in ax (Fig. 1). The cognitive mappings are expressed as "$(ax, bx) \rightarrow ay$", and "$(ax, bx) \rightarrow by$", respectively. These mappings are represented by the motion functions of A and B, denoted f_A, and f_B, respectively. Therefore, $f_A(ax, bx) = ay$, and $f_B(ax, bx) = by$. Denoting the state set of A as $\mathbf{A}$, and that of B as $\mathbf{B}$, cognizers A and B are identified with $(\mathbf{A}, f_A)$ and $(\mathbf{B}, f_B)$, respectively (f_A: $\mathbf{A} \times \mathbf{B} \to \mathbf{A}$, f_B: $\mathbf{A} \times \mathbf{B} \to \mathbf{B}$).

The state of the whole system shifts from (ax, bx) to (ay, by). Denoting $(f_A(ax, bx), f_B(ax, bx))$ as $(f_A, f_B)(ax, bx) \equiv F(ax, bx)$, where F is the motion function of the whole system (world), the above process is represented as $F(ax, bx) = (ay, by)$. Because the motion function determines uniquely one particular state, such as (ay, by), against a given state, such as (ax, bx), the system is deterministic with no probabilistic uncertainty about the behavior.

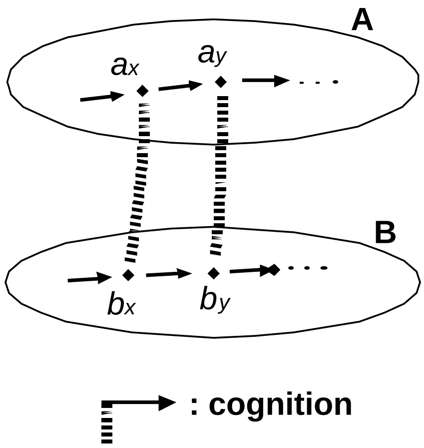

Fig. 1. Cognizers system composed of two cognizers, *A* and *B*. Each cognizer shifts a current state to another by cognizing the other. **A** and **B** are the state-set of *A* and *B*, respectively.

Key properties of cognition: selectivity and discriminability

There are two important aspects of cognition relevant to the present problem, selectivity and discriminability. As stated before, cognition is shifting from one state to another. This implies that cognition *selects* one particular state among many possibilities. In the above, *A* selects *ay*, and *B* selects *by*, among many possible succeeding states for each. This selection gives rise to a particular relation between the two cognizers, i.e. their state relation, such as (*a*y, *b*y). Therefore, selection by a cognizer affects the relation between the cognizer and the rest of the whole system, i.e. the environment, although the complete determination depends on the selection, not by one, but by both of them.

Discriminability is the ability to *discriminate* between different states of the environment. Discrimination is to act differently against different environmental states. Given a cognizer in a particular state (say *ax*), if it shifts from the same state (*ax*) to different states (say, *ay* and *ay*') against different environmental states (say, *ex* and *ex*') at different points of time, it is said that the cognizer in *ax* discriminates between the different environmental states, *ex* and *ex*'. Discrimination is closely related to the concept of causation, i.e. the cause-and-effect relation. By observing that an entity in the same state acts differently against different states of the environment, it can be understood that it is causally affected by something in the environment.

Definition of uncertainty of events

For an epistemologically omnipotent observer located outside the cognizers system, called the meta-observer, events occur without uncertainty because the state-transition of the system is deterministic as defined above. However, for cognizers, events can occur in an uncertain manner in this deterministic world due to the incomplete cognitive ability of cognizers.

In this scheme, cognizers are the subjects that experience events within the system. Uncertainty of events is defined as one-to-many correspondences between a given cognition and subsequent cognitions by a focal cognizer. For example, when one takes a ball from a black box, "taking out a ball" is the first cognition by a player, an internal participant in the system. It will be followed by a second cognition, which may be different depending on the trial, such as "a yellow ball" or "a red ball". Therefore, the degree of certainty, or the probability of an event is considered not in isolation, but in relation to the first cognition as a condition under which the next cognition or event occurs.

3 Uncertainties for cognizers, internal probability, and internal entropy

Mechanism by which uncertainty of events occurs to cognizers in deterministic system

It is required that a cognition as a trial is repeated in a cognizers system in question for formalizing the uncertainty of events to be meaningful, where uncertainty of events refers to one-to-many correspondences between a given cognition and subsequent cognitions by a focal cognizer.

A simple case can illustrate how such an uncertainty process occurs. Suppose the following state sequence of cognizer *A*, for example:

$$a1, a2, a3, \ldots, a1, a2, a4, \ldots \qquad (1)$$

In this sequence, *a1*, *a2* occurs twice. As before, cognition is formally represented as "$(ax, \bullet) \rightarrow ay$" for *A*, where • is a state of the environment, not specified. Therefore, it can say that cognition $(a1, \bullet) \rightarrow a2$ occurs twice.

Combining all cognizers existing in the environment together, and denote it as *E*, whose states are denoted as *ei* with a subscript *i* to distinguish different states; two-cognizers system shown in Fig. 1 is the case of *B* = *E*. By incorporating states of *E* into the sequence (1), the following sequence of the cognizers system is obtained:

$$(a1, e1), (a2, e2), (a3, e3), \ldots, (a1, e4), (a2, e5), (a4, e6), \ldots \qquad (2)$$

Uncertainty of events, or chance, for a focal cognizer is defined as different events occurring as a consequence of the same cognition when repeated. In the sequence (2), *A* experiences different environmental states, *e2* or *e5* after the cognition, (*a1*, •) $\rightarrow$ *a2*. After this cognition, different cognitions, (*a2*, *e2*) $\rightarrow$ *a3* or (*a2*, *e5*) $\rightarrow$ *a4*, occur, indicating a one-to-many correspondence between a given cognition and the consequent cognitions. This is the logical structure of uncertainty of events occurring to cognizers.

This uncertainty occurs because the cognizer does not discriminate between different states of the environment. In the above case, it means that *A* in *a1* does not discriminate between *E* in *e1* and *E* in *e4*; in other words, (*a1*, *e1*) $\rightarrow$ *a2*, and (*a1*, *e4*) $\rightarrow$ *a2*. In general, the fact that a given cognition does not discriminate between *n* different states of the environment means that the cognizer experiences them as the same event. And, chance or uncertainties arise if different events occur just after this same event.

Internal probability: probability of event occurring to cognizer

The above discussion suggests that imperfect discrimination produces the uncertainty of events occurring to cognizers. Next, this view needs to be developed more quantitatively to describe a quantity of uncertainty, i.e. the probability of an event for cognizers. In this theoretical scheme, the probability of an event means the probability of an event occurring to a focal cognizer after a given cognition, not to the external observer. In this sense it is internal, and, therefore, called internal probability [4].

Given a cognition (*a1*, *ei*) $\rightarrow$ *a2* by *A*, where, *ei* is one of *n* states of the environment, such as *e1*, *e2*, ..., *en*, and assume that each actually occurs when *A* is in *a1* at different points of time in the system under consideration. The environment also changes its state by cognition, such as (*a1*, *ei*) $\rightarrow$ *ei*', where *ei*' is a state of the environment (i = 1, 2, ..., n). Suppose *e1*' for *e1*, and *e2*' for *e2*, for simplicity. After the cognition (*a1*, *ei*) $\rightarrow$ *a2*, *A* faces *n* different environmental states, namely, *e1*', *e2*', ..., *en*', because it does not discriminate between *e1*, *e2*, ..., *en* —the reason for *n* states occurring after the non-discriminative cognition against *n* different states is due to the assumption of the deterministic, one-to-one mapping of the system. The cognizer, then, responds to those *n* different environmental states after the first cognition, (*a1*, *ei*) $\rightarrow$ *a2*. In the example shown in sequence (2), after the cognition (*a1*, •) $\rightarrow$ *a2*, *A* is going to face *e2* or *e5*.

Figure 2 shows a diagram of this cognitive process, in which {*e1*, *e2*, ..., *en*} = **Ex**, and {*e1*', *e2*', ..., *en*'} = **Ey**. **Ex** is the set of non-discriminated states of the environment for *A* in *a1*, and, **Ey** represents uncertainties that *A* is going to face. It must be said that *A* has the same cognitive image against the environment in n different states, $e1$, $e2$, ..., en, because it shifts to the same state, $a2$.

It should be noted that A in $a2$ does not necessarily discriminate every element from others belonging to **Ey**. How the cognizer in $a2$ behaves against **Ey** is another question. As shown in Fig. 2, assume that A shifts from $a2$ to $a3$ against $e1'$, $e2'$, and $e3'$, and from $a2$ to $a4$ against $e4$, $e5$, ..., en. This implies that two kinds of event can actually occur to the cognizer, because it gives rise to cognitions $(a2, \bullet) \rightarrow a3$, and $(a2, \bullet) \rightarrow a4$. Define the former cognition that occurs against $\{e1', e2', e3'\}$ as event 1, and the latter cognition that occurs against $\{e4', e5', ..., en'\}$ as event 2. Therefore, the probability (i.e. internal probability) of event 1 is given as $3/n$, and that of event 2 as $(n - 3)/n$. In other words, the probability of an event, 1 or 2, is obtained as the ratio of the number of environmental states giving rise to the cognition $(a2, \bullet) \rightarrow a3$, or $(a2, \bullet) \rightarrow a4$, to the total number of elementary events, n.

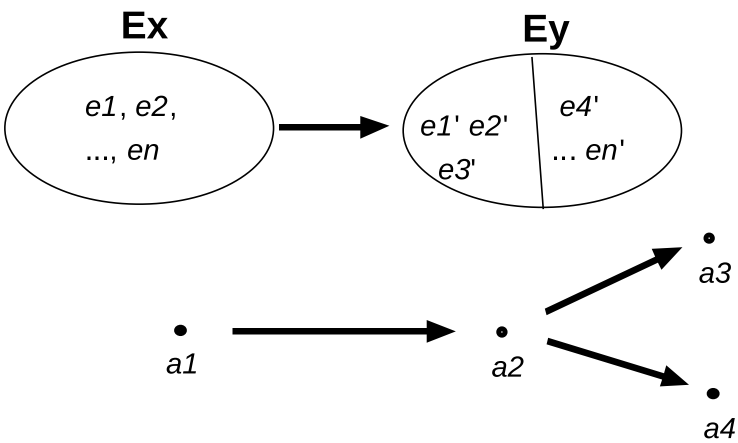

Fig. 2. Cognitive processes in which events occur to cognizer A in a manner of one-to-many correspondence. The number of elements (states) of **Ex** and of **Ey** are n (i.e. $|\mathbf{Ex}| = |\mathbf{Ey}| = n$).

Events can occur indeterministically to internal cognizers within a deterministic world, where probabilities of events are determined by inter-cognitive interaction between the cognizer and the environment. It should be noted that internal probability is determined neither by the cognizer nor by the environment, but by both. This fact can be easily understood: the probabilities of events 1 and 2 would change if the environment shifts its state differently from the above assumption. For example, if the environment shifts from $e1$ not to $e1'$, but to a different state, say $e1''$ in Fig 2, the probability of event 1 becomes $2/n$ in this case.

How to reduce the uncertainty of events

The above mechanism for describing the uncertainty of events suggests some possible ways a cognizer can reduce the uncertainty of events occurring. There are two different solutions to reduce the uncertainty for a cognizer: increase discriminability about different environmental states, by which *n* decreases, or, ignore the differences of the environmental states in the second cognition. For example, in Fig. 2, if the cognizer does not discriminate between every element from others belonging to **Ey**, i.e., shifting from *a2* to *a3* for every states of **Ey**, it would not experience any uncertainty, implying only unique event occurs to it.

Living systems appear to use both strategies. It is widely observed that living entities or their components discriminate between different states of their environment and respond appropriately to maintain particular relationships with others. At the molecule and the cell level, molecular recognition functions as a very important part of self-organization and regulation, and at the organism level cognitive processes in neural networks produce adaptive behaviors of animals [6-8].

On the other hands, living systems sometimes ignore differences in their environment to maintain their organizational order. For example, a cell is semi-isolated from the outside by the cell membrane or cell wall, by which they can ignore many different environment occurrences. Dormant spores do not discriminate between many events or stimuli happening in the environment, physical or chemical, and can resist a wide range of temperature or pressure. Generalist organisms can use a wide range of resources and are tolerant to a variety of physical and biological conditions, and, therefore, they can survive a wider range of environmental conditions. By this ability, generalist organisms may survive better in an uncertain environment than specialists [9].

Internal entropy

The above discussion suggests that cognition, i.e. sensation-and-action, affects types of event and their probabilities, which implies that cognition produces an event-environment, subject-dependent environment, characterized by a particular set of events and the probability distribution specific to the cognizer. Uexküll [10] sees every animal as a subject, which selects stimuli from general influences of the outer world, and responds to them in a certain way. These responses, in turn, consist of certain effects on the outer world, which again influence the stimuli. Thus, sensation and action creates a subject-dependent surrounding world, i.e. *umwelt*. This view can be extended to living entities in general, and even to abiotic, physical ones.

As discussed in Fig. 2, the cognition $(a1, \bullet) \rightarrow a2$ by A gives event 1 at the probability of $3/n$, and event 2 at the probability of $(n - 3)/n$. Based on the internal probabilities of events followed by a given cognition, a quantity for measuring disorder of the environment occurring after a given cognition can be introduced. For such an index, the formula of Shannon's entropy may be available. Therefore, the internal entropy of the environment for a given cognition can be defined as $-\sum_{i=1}^{n} p_i \log_2 p_i$, where, pi indicates the internal probability of event i occurring after the cognition. In the case of Fig 2, the internal entropy for cognition $(a1, \bullet) \rightarrow a2$ is given as $\frac{3}{n}\log_2\frac{n}{3}+\frac{n-3}{n}\log_2\frac{n}{n-3}$. This quantity indicates the amount of uncertainty about events occurring to the cognizer by taking a particular cognition.

Every moment, living systems, such as cells and organisms, are continually taking cognitions against a current situation. This quantity given to each cognition may provide an estimate of total uncertainty of a subject-dependent environment formed by inter-cognition between the subject cognizer and the environment. Internal entropy does not necessarily increase with time. For example, a room does not necessarily become disordered as time goes by according to the second law of thermodynamics. On the contrary it may increase or keep its current order, if an owner occasionally tidies the room. The room is a part of the environment of the owner, which is formed by cognitive interactions between the owner and things in the room. The order of the room should be measured in terms of internal entropy, not ordinary entropy viewed from the external observer.

Internal probability and internal entropy may explain the problem concerning just where the initial low-entropy states of thermodynamic systems come from, such as a gas held by the partition in a box, a lump of sugar held over a coffee cup, etc. [11]. Systems in low-entropy states are possible because those thermodynamic situations are part of the environment of the people, who produce or build them by cognitive interactions as discussed above. In this sense, entropy of such systems should be realized as internal entropy, which measures the order of the environment under interaction with the subjects who cognizes it.

4 Conclusion

In general, reduction of uncertainty may be associated with information concept, that is, the more information known about an object reduces the uncertainty of it [12]. Accordingly, the uncertainty of events, or chance, occurs due to ignorance about the object. It is important to identify the knowing subject, an information receiver, whether it is the external observer or a material participant (i.e. cognizer) in the system. Traditionally, the problem concerning how information reduces the uncertainty of events has been treated situationally where the subject is an epistemic, external observer; even if it is the case of an internal participant, knowing is treated as epistemic processes, rather than material movement occurring in the system. For that observer, information may reduce the uncertainty of event happening there.

However, in this paper, the approach is from the viewpoint of a material participant in a system, which cognizes, i.e. senses and acts, and the experience of the uncertainty of events. To this end, a theoretical scheme, in which the subject of knowing or the information receiver is extended to every material participant in a system, is introduced. In this sense, internal probability is different from Bayesian probability [4]. It can be understood that our world is our environment, formed dependently by our cognitive interactions through experiments, observations, and acting with other material entities, just as the room for the owner is so.

5 References

1. Schrödinger, E. *What is Life?*, reprinted in *What is Life? with Mind and Matter and Autobiographical Sketches*, Cambridge University Press, Cambridge, 1944.
2. Nakajima, T. Biological probability: cognitive processes of generating probabilities of events in biological systems. *J. Theor. Biol.* **1999**, 200, 77-95.
3. Nakajima, T. Is the world deterministic? mental algorithmic process generating external reality by inverse causality. *Int. J. Gen. Sys.* **2001,** 30, 681-702.
4. Nakajima, T. Probabilities of encounters between objects in biological Systems 2: cognizer view. *J. Theor. Biol.*, **2003**, 221, 39-51.
5. Laplace, M. A philosophical essay on probabilities; reprint in 1951, Dover, publications, inc., NW, 1814.
6. Albert, b., Bray, D., Lewis, J., Raff, M., Roberts, K., and Watson, J. *Molecular Biology of the Cell* (3rd ed.). New York: Garland Publishing, Inc. 1994.
7. Lerner, R. A. & Tramontano, A. Catalytic Antibodies. *Sci. Am.* **1988**, 258, 58-70.

8. Root-Bernstein, R. S. and Dillon, P. F. Molecular complementarity I: the complementarity theory of the origin and evolution of life. *J. Theor. Biol.* **1997**, 188, 447-479.
9. Pianka, E. R. On *r* and *K* selection. *Amer. Natur*. **1970**, 102, 592-597.
10. Uexküll, J. von. *Theoretical Biology*, New York: Harcourt, Brace. 1926.
11. Penrose, R. The emperor's new mind. Oxford University Press. NY, 1989.
12. Klir, G. J. and Wierman, M. J. *Uncertainty-based information*. Physica-Verlag, Hidelberg, 1999.