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Information World.

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Abstract: The long term discussion of the concept of information and its relationship to the entropy involves people of different research areas mathematics, engineering, physics, chemistry, biology, linguistic, philosophy, theology etc. The primary idea of the *Information World* is introduced and considered from the philosophical rather than mathematical point of view. It is argued that the concept of *information* could be a fundamental one, rather than the derivative.

Keywords: entropy; information; density matrix

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1 Motivation

The primary reason for the current article is a widespread use of the term "information" in different instances, without actually defining what that means in the particular context and how to measure its value. Obviously, this is not always possible or simply reasonable to do. Usually, it is accepted by default what kind of information is under consideration. The information theory has numerous examples of how definitions could be done. The fact is, the information is used everywhere, where we have "events" to study and measure. The events frequently associated with some random variables or sequences that imply applicability of some kind of statistics.

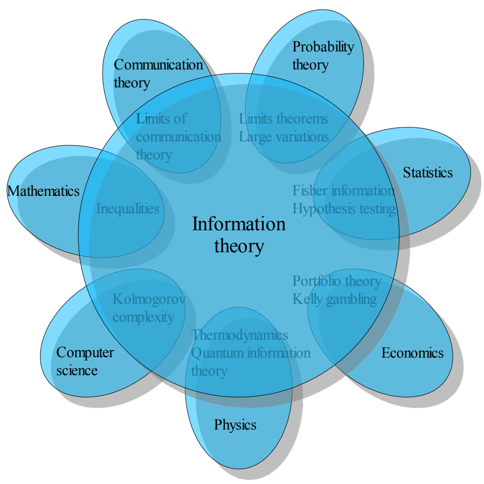


Figure 1: The relationship of information theory with other fields (after T.M.Cover and J.A.Thomas [7] with little modifications).

The information concept has direct relation to physical and statistical concept of entropy (sometimes the information called *neg entropy*). All primary properties of the entropy follows directly from the properties of the $\log(x)$ function. This was used by Claude Shannon in his communication theory and as a matter of fact was implicitly used in information context far more years before by musicians in logarithmic encoding sound into octaves of notes (see e.g. [11])
The current presentation is the attempt to express a view to the problem in the intuitive and philosophical sense rather than in strict mathematical theoretical arguments. Nevertheless, the artifacts used in the article were extracted from the mathematical formulations of the information theory and physical (especially quantum) understanding of information. In the next section the concept of information world is introduced to justify the appearance and discovery of new events carrying information in (into) the real physical world. This should not be considered as alternative or complementary to unification of information by R.Frieden [3, 4] or to mathematical theory of information by J.Kahre [6]. The article mostly concern the foundations of science in the frame

of information theory. However, the current research is directed to achieve a generalized common viewpoint.

2 Classic Information Theory

The classic theory of information is based on logarithmic characteristics of some density mass function $f(x)$ of the probability distribution of event x. For example, Shannon's entropy [1, 2]

$$H(f(x)) = -\int_{-\infty}^{\infty} f(x) \log f(x) dx$$

or Fisher information [9]

$$I(f(x)) = \int_{-\infty}^{\infty} \frac{[\partial f(x)/\partial x]^2}{f(x)} dx$$

share a similar behavior (in fact they differ by a negative factor for a Gaussian distribution function). In the vicinity of $f(x) = 0$ the both $\log f(x)$ and $1/f(x)$ have singularity. This simple observation usually ignored due to continuity agreements $x \log x \rightarrow_{x \rightarrow 0} 0$. Let us ask the question: what it means the $x \rightarrow 0$?
In the discrete case the answer is more obvious. The entropy is

$$H(p) = -\sum_x p(x) \log p(x)$$

where $p(x) = \lim_{N_{tot} \rightarrow \infty} \frac{N_{occur}}{N_{tot}}$ is a probability of occurrence of event x. The Shannon's communication theory is build on the suggestion that $p(x)$ is known for every event x. Let us ask the questions: what is the event x? how does one know that it will occur? how to discover a characteristic measure (or measurement units of x) reflecting the event existence?
Turning now to the foundation of science one could say that the scientific discovery consists of (1) an "experiment" discovery, allowing the measurement process, and (2) an "event" discovery, which provides the "value" of the event measurement and, as a consequence, the probability measure. This article is not about measurement theory. The only note from the last observation is important

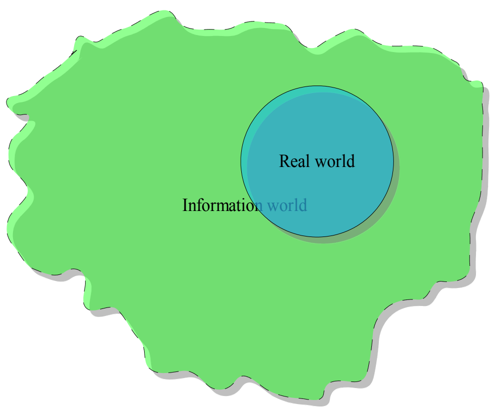


Figure 2: The Information world as an environment of the Real world

here, that the information content (whatever it is) in classical information theory can be estimated for "known" events only. And what about things we don't know yet? How much information is in the events from "out there"? Let us avoid such a theological superficial notes by introducing a term "Information world" for all what has an information content as a complementary to "Real world" where a measurement and information processing into mankind knowledge is theoretically possible. A simple picture is where the "Real world" is embedded into "Information world" (see Fig.2). The boundary between the real world and environmental information world can be seen similar to vacuum artifact in quantum theory.

3 Quantum Information Theory

Information content of the quantum system can be introduced through the density matrix operator $\hat{\rho}$ found as a solution of Liouville-von Newmann evolution equation (see e.g. [8])

$$\frac{\partial\hat{\rho}}{\partial t} = \frac{1}{i\hbar}\left[\hat{H}\hat{\rho}\right]$$

where $\hat{H}$ is the Hamiltonian of a system. If the Hamiltonian does not explicitly depend on time t the corresponding evolution operator of the system

$$\hat{U}(t) = e^{-i\hat{H}t/\hbar}$$

[1]

defines a unitary evolution of the density matrix

$$\hat{\rho}(t) = \hat{U}(t)\hat{\rho}(0)\hat{U}^{\dagger}(t)$$

The entropy of the quantum system is

$$S(\hat{\rho}) = -\mathtt{Tr}(\hat{\rho}\log\hat{\rho})$$

and the unitary evolution does not change trace, eigenvalue spectrum, or entropy of the density matrix. This means that information cannot be created or erased, just rearranged. This is valid for the universe as a whole and does not required for subsystem of our Real world. This is what could be called the Information world that contains the whole information.
It could be interesting to consider the interaction of the environment Information world with the Real world in an analogy with the density matrix renormalization group numerical approach (for review of DMRG method see e.g. [12]). Consider two parties: the *system* and the *environment* (the rest of the World) with corresponding state vectors $|s\rangle$ and $|e\rangle$ with density matrices $\hat{\rho}^S$ and $\hat{\rho}^E$. The state vector of the whole World is tensor product

$$|\psi\rangle = |s\rangle \otimes |e\rangle \tag{1}$$

and the trace over system gives a *reduced* density matrix of the environment

$$\mathtt{Tr}_S\hat{\rho} = \hat{\rho}^E \tag{2}$$

[1]the exponential operator can be defines as a power series expansion $e^{-i\hat{H}t/\hbar} = \hat{I} - \frac{it}{\hbar}\hat{H} + \frac{1}{2!}\left(\frac{it}{\hbar}\right)^2\hat{H}\hat{H} + ...$ or as function operator spectral representation $f(\hat{X}) = \sum_n f(x_n)\,|n\rangle\,\langle n|$ using eigenvalues and eigenvectors of the operator $\hat{X}\,|n\rangle = x_n\,|n\rangle$ in Dirac bra-ket notations.

while the trace over environment gives reduced density matrix of the system

$$\mathtt{Tr}_E\hat{\rho} = \hat{\rho}^S \tag{3}$$

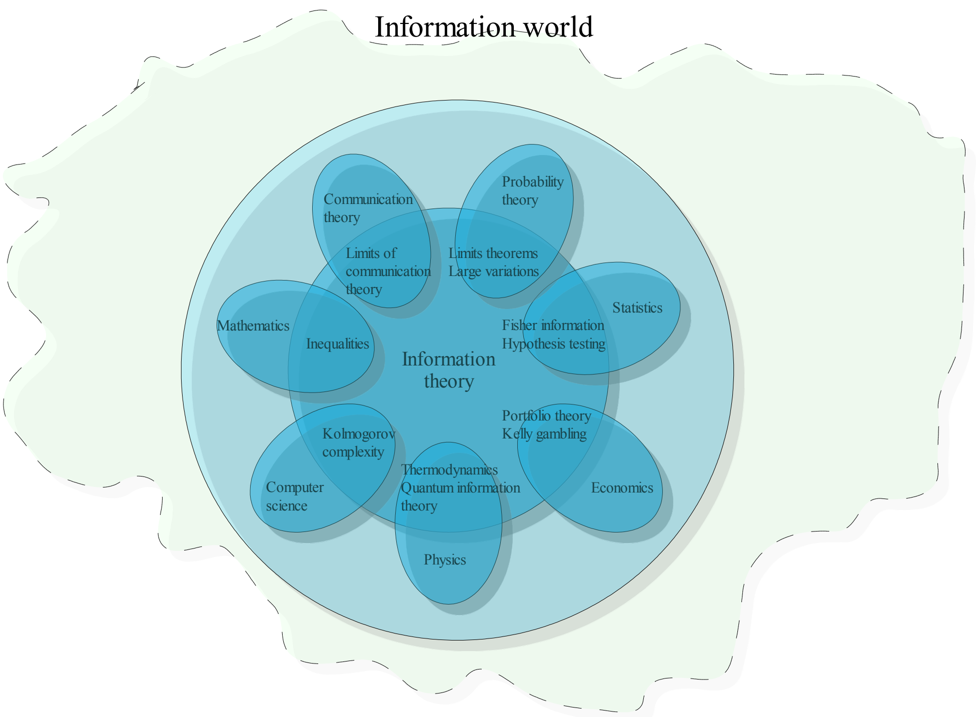
The unitary evolution can nor create or destroy information, it can move it between subsystems because for any unitary operator (describing a measurement) $\hat{U}$

$$\mathtt{Tr}_S(\hat{U}\hat{\rho}^S) = \mathtt{Tr}_S(\hat{U}\mathtt{Tr}_E\hat{\rho}) = \mathtt{Tr}((\hat{U}\otimes\hat{I}_E)\hat{\rho}) \tag{4}$$

From the above considerations one can conclude that the amount of information in the World is determined by the number of observable (or measurable) subsystems, which interact with each other and transfer information between each other.

4 Conclusion

In the present article the Information world is introduced rather intuitively and simple intuitive arguments were given as classical and quantum information theoretical artifacts. The following philosophical picture can conclude the discussion and [2] open ... a great edge in debates because nobody really knows what information is.



[2]by paraphrasing Richard P. Feynman note [10] that Shannon adopted the term *entropy* on the advice of John von Newmann, who declared that it would give him "... a great edge in debates because nobody really knows what entropy is anyway" [5]

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