Design of Similarity and Dissimilarity Measures for Fuzzy Sets on the Basis of Distance Measure

Sang-Hyuk Lee, Witold Pedrycz, and Gyoyong Sohn

Abstract

In this paper, we survey the relationship between the similarity measure and dissimilarity measure for fuzzy sets. First, we design a similarity measure using a distance measure for fuzzy sets and prove its usefulness. From this result, we assert that the similarity between two complementary fuzzy sets satisfies the fuzzy entropy definition. We also show that the summation of the similarity and dissimilarity measures between two membership functions of fuzzy sets constitute all the information of the fuzzy set itself. We then extend our results to two data group fuzzy sets. Data similarity and dissimilarity measures between two fuzzy membership functions satisfy complementary. We also verify and discuss the characteristics of the relation between the similarity measure and dissimilarity measure with illustrative example.

Keywords: Similarity measure, distance measure, fuzzy entropy.

1. Introduction

The similarity or dissimilarity between two data sets is commonly measured by statistical analyses, i.e., on the basis of average values or standard deviations. These approaches present information from different perspectives and are therefore sometimes at odds with the heuristic point of view. In order to analyze ambiguous data, we must consider the data set as a fuzzy set with a degree of membership. The analysis of similarity and dissimilarity is essential to the complete study of the data or information in fuzzy sets. The degree of similarity between two or more data sets plays a key role in the fields of decision-making, pattern classification, etc. [1-6]. Numerous researchers have explored the design of similarity measures [6-10], which is easily achieved with a fuzzy number. Such designed similarity measures, however, are restricted to triangular or trapezoidal membership functions [6-9]. Similarity measures that are based on distance measures are more broadly applicable to general fuzzy membership functions, including even the non-convex fuzzy membership functions [9].

The determination of the similarity measure and dissimilarity measure for a data group is an area of great interest. If the similarity measure of a data group is represented, then it is also possible to represent the dissimilarity. Basically, a high degree of similarity data indicates a low degree of dissimilarity. Hence, we also surveyed the relationship between the similarity measure and the dissimilarity measure. For a given data group, in order to determine the similarity and dissimilarity we must compare two sets of data. One is a deterministic data set and the second is the complementary data set. Similarity and dissimilarity measures are constructed by applying the distance measure to the two data sets. The dissimilarity measure for the two data sets can be regarded as the distance between them. The similarity measure is designed by calculating the common area or overlap between the two fuzzy membership functions.

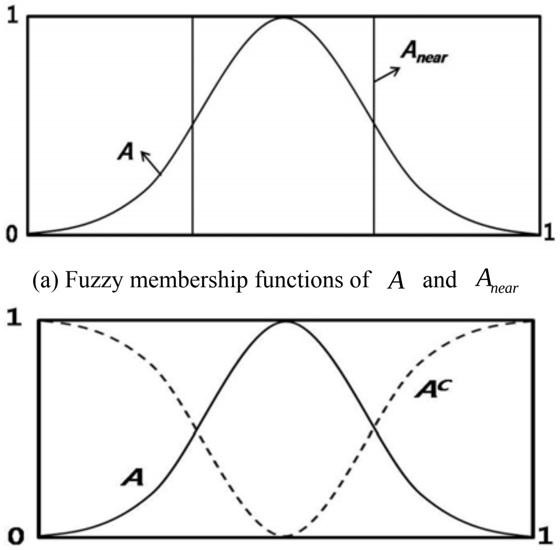
The correlation between similarity and dissimilarity has been explored from various viewpoints [11]. Besides a physical explanation, Liu proposed a relationship between the distance and similarity measures: his paper indicates that the summation of the distance value and similarity value constitute the totality of information [10]. In this paper, we analyze the relationship between similarity and dissimilarity for a given data group and its corresponding numeric data set or complementary data set. The totality of the relational information held by the data can be represented in terms of just the similarity and dissimilarity values. With the help of the distance measure, we can design the similarity measure. The similarity measure thereby obtained can be used to calculate the dissimilarity measure on the basis of the total data summation property. In the following chapter, we discuss the similarity measure and dissimilarity measure of two fuzzy membership functions. We also introduce the previously obtained fuzzy entropy and similarity measure. In Chapter 3, we present the procedure whereby the dissimilarity measure is obtained from the similarity measure, and vice versa. A simple example of this is illustrated in Chapter 4. Our

Corresponding Author: Gyoyong Sohn is with the Department of Computer Science, Chungbuk National University, #81 Gaesin-dong, Cheongju, Republic of Korea.
E-mail: gysohn74@hanmail.net
Manuscript received 3 Nov. 2008; revised 18 June 2009; accepted 19 June. 2009.

conclusions then follow in Chapter 5.

2. Similarity and Dissimilarity Measure Analysis

In this chapter, we survey the relationship between the similarity and dissimilarity measure of a given data set with respect to the corresponding comparative data set. We also consider the fuzzy membership function and corresponding membership function in the design of similarity and dissimilarity measures. Fuzzy membership function pairs are illustrated in Fig. 1.



(a) Fuzzy membership functions of A and A_{near}

(b) Fuzzy membership functions of A and A^C

Fig. 1. Fuzzy membership function pairs.

Similarity measure studies are all concerned with the design of a similarity measure, which is then applied to the computation of degree of similarity on the basis of a distance measure. The suggested definitions of similarity measure have the same meaning for two data groups or fuzzy sets being compared. Liu proposed a definition of the axiomatic similarity measure in his paper [10]. The similarity measure for $\forall A,B \in F(X)$ and $\forall D \in P(X)$ has four properties, which are as follows:

(S1) $s(A,B) = s(B,A)$, $\forall A,B \in F(X)$

(S2) $s(D,D^c) = 0$, $\forall D \in P(X)$

(S3) $s(C,C) = \max_{A,B\in F} s(A,B)$, $\forall C \in F(X)$

(S4) $\forall A,B,C \in F(X)$, if $A \subset B \subset C$, then $s(A,B) \ge s(A,C)$ and $s(B,C) \ge s(A,C)$

where $F(X)$ is a fuzzy set and $P(X)$ is a numeric set. The similarity measure between A and A_{near} is proposed in Theorem 2.1. We verify its usefulness by proving it.

Theorem 2.1: For $\forall A \in F(X)$ and crisp set A_{near} in Fig. 1 (a),

$$s(A,A_{near}) = d(A \cap A_{near},[0]_X) + d(A \cup A_{near},[1]_X) \qquad (1)$$

is the similarity measure, where A_{near} is one when $\mu_A(x) \ge 0.5$, and is otherwise zero. $[0]_X$ and $[1]_X$ are fuzzy sets in which the value of the membership functions are zero and one, respectively. The Hamming distance is defined by $d(A \cap A_{near},[0]_X) = \frac{1}{n}\sum_{i=1}^{n} |\mu_{A\cap A_{near}}(x_i) - 0|$. $d(A \cup A_{near},[1]_X)$ is also obtained similarly.

Proof: (S1) is clear from (1) itself, and for crisp set D, it is clear that $s(D,D^C) = 0$. Hence, (S2) is satisfied. It is clear that (S3) indicates that the similarity measure of the exact two fuzzy sets $s(C,C)$ satisfies the maximum value among various similarity measures with different fuzzy sets A and B, because $d(C \cap C,[0]_X)$ + $d(C \cup C,[1]_X)$ represents the entire region of Fig.1(a). Finally (S4) is proved. If $A \subset A_{1near} \subset A_{2near}$, then $d(A \cap A_{1near},[0]_X) \ge d(A \cap A_{2near},[0]_X)$ and $d(A \cup A_{1near},[1]_X) \ge d(A \cup A_{2near},[1]_X)$. It naturally follows that

$$s(A,A_{1near}) = d(A \cap A_{1near},[0]_X) + d(A \cup A_{1near},[1]_X)$$
$$\ge d(A \cap A_{2near},[0]_X) + d(A \cup A_{2near},[1]_X) = s(A,A_{2near}).$$

Similarly, $s(A_{1near},A_{2near}) \ge s(A,A_{2near})$ is satisfied by the inclusion properties of $d(A_{1near} \cap A_{2near},[0]_X) \ge d(A \cap A_{2near},[0]_X)$ and $d(A_{1near} \cup A_{2near},[1]_X) \ge d(A \cup A_{2near},[1]_X)$. ■

This similarity (1) indicates the common areas of the two membership functions. For Fig.1 (b), the similarity measure between fuzzy sets A and A^C is also satisfied by (2) similarly as with (1).

$$s(A,A^C) = d(A \cap A^C,[0]_X) + d(A \cup A^C,[1]_X) \qquad (2)$$

It is logical that there should be numerous expressions that satisfy the similarity definition. Proof is obtained as it was for Theorem 2.1. We proposed the following similarity measure for two arbitrary fuzzy sets in our previous work [9, 12]:

For any set $A,B \in F(X)$, if d is the Hamming distance, then

$$s(A,B) = 1 - d\left(\left(A \cap B^C\right),[0]_X\right) - d\left(\left(A \cup B^C\right),[1]_X\right) \qquad (3)$$

and $s(A,B) = 2 - d\left(\left(A \cap B\right),[1]_X\right) - d\left(\left(A \cup B\right),[0]_X\right)$ (4)

are also similarity measures for sets A and B.

In (3) and (4), fuzzy set B can be replaced by A_{near} and A^C.

In Fig. 1 (b), the similarity measure between A and A^C is defined as $s(A,A^C)$. We can now discuss the meaning of $s(A, A^C)$.

$$s(A, A^C) = d(A \cup A^C, [1]_X) + d(A \cap A^C, [0]_X) \quad (5)$$

$s(A, A^C)$ represents fuzzy entropy; this is proven by verifying the properties of fuzzy entropy. We first use Liu's definition of fuzzy entropy, after which proofs follow in which the properties (E1)–(E4) are verified [10].

(E1) $e(D) = 0, \forall D \in P(X)$;

(E2) $e([1/2]_X) = max_{A \in F(X)} e(A)$;

(E3) $e(A^*) \le e(A)$, for any sharpening A^* of A;

(E4) $e(A) = e(A^C), \forall A \in F(X)$;

where $[1/2]_X$ is the fuzzy set in which the value of the membership function is $1/2$.

For all crisp sets D,

$$s(D, D^C) = d(D \cup D^C, [1]_X) + d(D \cap D^C, [0]_X),$$
$$= d([1]_X, [1]_X) + d([0]_X, [0]_X) = 0.$$

Hence, (E1) is satisfied. For (E2),

$$s([1/2]_X, [1/2]_X{}^C) =$$
$$d([1/2]_X \cup [1/2]_X{}^C, [1]_X) + d([1/2]_X \cap [1/2]_X{}^C, [0]_X)$$
$$= d([1/2]_X, [1]_X) + d([1/2]_X, [0]_X) = 1$$

is maximum. (E2) is also satisfied. We now must prove (E3).

$$s(A^*, A^{*C}) = d(A^* \cup A^{*C}, [1]_X) + d(A^* \cap A^{*C}, [0]_X)$$
$$\le d(A \cup A^C, [1]_X) + d(A \cap A^C, [0]_X) = s(A, A^C),$$

where A^* is greater than A when $\mu_A(x) \ge 1/2$, and $A^* \le A$ when $\mu_A(x) \le 1/2$. Finally, (E4) is satisfied easily from (5) itself. ■

From Fig. 1, the relationship between the similarity and dissimilarity measures for fuzzy set A with respect to A_{near} or A^C can be explained by the total area. The total area is one (universe of discourse × maximum membership value = $1 \times 1 = 1$); this represents the totality of information in the set. Hence, the totality of information consists of the similarity measure and dissimilarity measure as follows:

$$s(A, A_{near}) + D(A, A_{near}) = 1 \quad (6)$$

$$s(A, A^C) + D(A, A^C) = 1 \quad (7)$$

Through a comparison of Figs. 1(a) and (b) and Eqs. (4) and (6), we obtain the following proposition.

Proposition 2.1: $D(A, A_{near})$ represents the dissimilarity measure between fuzzy sets A and A_{near}.

$$D(A, A_{near}) = d\left(\left(A \cap A_{near}\right), [1]_X\right) + d\left(\left(A \cup A_{near}\right), [0]_X\right) - 1$$

With similarity measure (1), the similarity measure between fuzzy set A with respect to the corresponding crisp set A_{near} can be also formulated. The following theorem proves (6), which represents the relationship between similarity and dissimilarity measures.

Theorem 2.2: The total information in fuzzy set A and the corresponding crisp set A_{near}, which is equal to the summation of similarity and dissimilarity measure.

$$s(A, A_{near}) + D(A, A_{near})$$
$$= d(A \cap A_{near}, [0]_X) + d(A \cup A_{near}, [1]_X)$$
$$+ d\left(\left(A \cap A_{near}\right), [1]_X\right) + d\left(\left(A \cup A_{near}\right), [0]_X\right) - 1, \quad (8)$$

is equal to one.

Proof: (8) says that the summation of similarity measure and dissimilarity measure is equal to one, the total region in Fig.1 (a). In (8),

$$d(A \cap A_{near}, [0]_X) + d\left(\left(A \cap A_{near}\right), [1]_X\right) = 1 \text{ and}$$
$$d(A \cup A_{near}, [1]_X) + d\left(\left(A \cup A_{near}\right), [0]_X\right) = 1.$$

Hence, $s(A, A_{near}) + D(A, A_{near}) = 1 + 1 - 1 = 1$. ■

Similarity measure (7) can be proved similarly. It is thus made clear that the total information of fuzzy set A can be represented by in terms of the similarity and dissimilarity measures.

Proposition 2.2: Following from Proposition 2.1, the dissimilarity between A and A^C is

$$D\left(A, A^C\right) = d\left(\left(A \cap A^C\right), [1]_X\right) + d\left(\left(A \cup A^C\right), [0]_X\right) - 1.$$

Hence it is logical that the region not common to both represents the dissimilarity between two fuzzy sets A and B, as follows:

$$D(A, B) = d(A, A \cap B) + d(B, A \cap B).$$

We can now propose a theorem concerning similarity and dissimilarity measures.

Theorem 2.3: Total information in fuzzy sets A and B, that is, the summation of similarity and dissimilarity, is

$$s(A, B) + D(A, B) = d(A \cap B, [0]_X) + d(A \cup B, [1]_X)$$
$$+ d(A, A \cap B) + d(B, A \cap B) = 1. \quad (9)$$

Proof: (9) shows that the summation of similarity and dissimilarity is equal to one. In (9),

$$d(A, A \cap B) = \frac{1}{n} \sum_{i=1}^{n} | \mu_A(x_i) - \mu_{A \cap B}(x_i) |$$

and
$$d(A \cap B, [0]_X) = \frac{1}{n} \sum_{i=1}^{n} | \mu_{A \cap B}(x_i) - 0 |, \quad (10)$$

respectively. $d(A, A \cap B)$ represents the distance between $\mu_A(x_i)$ and $\mu_{A \cap B}(x_i)$, $\forall x_i \in X$. Furthermore, $d(A \cap B, [0]_X)$ denotes the distance between $\mu_{A \cap B}(x_i)$ and zero. In (9), all membership functions are defined in the same universe of discourse. Hence, it is logical that $d(A, A \cap B) + d(A \cap B, [0]_X)$ represents $d(A, [0]_X)$.

Next,

$$d(B, A \cap B) = \frac{1}{n} \sum_{i=1}^{n} | \mu_B(x_i) - \mu_{A \cap B}(x_i) |$$

and
$$d(A \cup B, [1]_X) = \frac{1}{n} \sum_{i=1}^{n} | \mu_{A \cup B}(x_i) - 1 |. \quad (11)$$

In (11), $d(B, A \cap B)$ is the distance between B and the minimum value among A and B. If

$\mu_A(x_i) \geq \mu_B(x_i)$, $d(B, A \cap B)$ is equal to zero because it is the distance between B and itself. Otherwise, it is the distance between B and A. In this case, $\mu_A(x_i) \leq \mu_B(x_i)$. Finally, $d(A \cup B, [1]_X)$ represents the distance between one and the maximum value among A and B. Therefore, the following equation is satisfied:

$$d(B, A \cap B) + d(A \cup B, [1]_X) = d(A^C, [1]_X) \qquad (12)$$

Then,

$$d(A \cap B, [0]_X) + d(A \cup B, [1]_X) + d(A, A \cap B) + d(B, A \cap B)$$
$$= d(A, [0]_X) + d(A^C, [1]_X) = 1.$$

This result means that the summation of similarity and dissimilarity measures equals the whole region—the entirety of the information in fuzzy sets A and B.

3. Derivation of Dissimilarity and Similarity Measure

Liu insisted that entropy can be calculated from the similarity measure and distance measures, which are denoted by $e < s >$ and $e < d >$ [10]. We have constructed the similarity measures with distance measure d, those are Eqs. (1) to (4). In Liu's results, $s + d = 1$, where d is the dissimilarity measure, which makes it logical to obtain following result:

$$D(A, B) = d(A, A \cap B) + d(B, A \cap B) = 1 - s(A, B)$$

Therefore, we propose that the similarity measure

$$s < d >= 1 - d(A, A \cap B) - d(B, A \cap B) \qquad (13)$$

satisfies the relation $s = 1 - d$.

At this point, it is interesting to explore whether (13) represents the similarity measure.

Proof: (S1) is clear from (13). Furthermore, $s < d >= 1 - d(D, D \cap D^C) - d(D^C, D \cap D^C)$ is zero because $d(D, D \cap D^C) + d(D^C, D \cap D^C)$ satisfies $d(D, [0]_X) + d(D^C, [0]_X) = 1$. Hence, (S2) is satisfied. (S3) is also satisfied by $d(C, C \cap C) + d(C, C \cap C) = 0$; it is logical that $s(C, C)$ is maximal. Finally, (S4) states $1 - d(A, A \cap B) - d(B, A \cap B) \geq 1 - d(A, A \cap C) - d(C, A \cap C)$ because $d(A, A \cap B) =$ $d(A, A \cap C)$ and $d(B, A \cap B)$ $\leq d(C, A \cap C)$ are satisfied. Similarly, $s(B, C) \geq s(A, C)$ is satisfied. Therefore, the similarity measure can be obtained from the dissimilarity measure by (13). ■

With (13), we learn yet another fact concerning similarity and dissimilarity measures. Liu proposed a relationship between entropy and similarity measure in Propositions 3.5 and 3.6 in reference [10]. With Liu's property of a one-to-one correspondence between similarity and distance, distance means the dissimilarity between two groups of data. We have derived a similarity measure with the distance measure. Furthermore, with the similarity measure, we also derived the fuzzy entropy.

Next, the dissimilarity measure $D(A, B)$ can be obtained from the similarity measure? Similarity measure (13) is an obvious form that can be converted into the dissimilarity measure using the relation $s + d = 1$. We can now verify the dissimilarity derivation by means of our similarity measures (1), (3), and (4). By this relationship, the dissimilarity measure can be obtained.

$$D(A, B) = 1 - d(A \cap B, [0]_X) - d(A \cup B, [1]_X), \qquad (14)$$

$$D(A, B) = d(A \cap B^C, [0]_X) + d(A \cup B^C, [1]_X), \qquad (15)$$

and $D(A, B) = d(A \cap B, [1]_X) + d(A \cup B, [0]_X) - 1. \qquad (16)$

These dissimilarity measures stand for the distance between fuzzy sets A and B. By Lis's definition of the distance measure [10],

(D1) $d(A, B) = d(B, A)$, $\forall A, B \in F(X)$;

(D2) $d(A, A) = 0$, $\forall A \in F(X)$;

(D3) $d(D, D) = \max_{A, B \in P} d(A, B)$, $\forall D \in P(X)$;

(D4) $\forall A, B, C \in F(X)$, if $A \subset B \subset C$, then $d(A, B) \leq d(A, C)$ and $d(B, C) \leq d(A, C)$.

Dissimilarity measure (14) is easily verified as follows.

(D1) is clear from (14) itself. For (D2),

$$D(A, A) = 1 - d(A \cap A, [0]_X) - d(A \cup A, [1]_X)$$
$$= 1 - d(A, [0]_X) - d(A, [1]_X) = 0.$$

(D3) is proved easily:

$$D(G, G^C) = 1 - d(G \cap G^C, [0]_X) - d(G \cup G^C, [1]_X)$$
$$= 1 - d([0]_X, [0]_X) - d([1]_X, [1]_X) = 1.$$

Finally, for $A \subset B \subset C$,

$$D(A, B) = 1 - d(A \cap B, [0]_X) - d(A \cup B, [1]_X)$$
$$\leq 1 - d(A \cap C, [0]_X) - d(A \cup C, [1]_X) = D(A, C)$$

is satisfied because $d(A \cap B, [0]_X) = d(A \cap C, [0]_X)$ and $d(A \cup B, [1]_X) \geq d(A \cup C, [1]_X)$. Similarly, it is also clear that $D(B, C) \leq D(A, C)$. This verification shows that the construction of a distance measure or dissimilarity measure between two fuzzy sets is possible by means of the similarity measure. From the same verification, (15) and (16) also represent the distance between two different fuzzy sets A and B.

4. Illustrative Example

Let us consider the next fuzzy set with membership function

$$A = \{x, \mu_A(x)\}$$
$$= \{(0.1, 0.2), (0.2, 0.4), (0.3, 0.7), (0.4, 0.9), (0.5, 1), (0.6, 0.9), (0.7, 0.7), (0.8, 0.4), (0.9, 0.2), (1, 0)\}.$$

It is a data group that has ten points of information with a membership value. Then the complement of A is written as

$A^C = \{x, \mu_{A^C}(x)\}$
$=\{(0.1,0.8), (0.2,0.6), (0.3,0.3), (0.4,0.1), (0.5,0), (0.6,0.1), (0.7, 0.3), (0.8,0.6), (0.9,0.8), (1,1)\}$.

A_{near} can be assigned various variables. For example, the value of crisp set $A_{0.5}$ is one when $\mu_A(x) \ge 0.5$, and is zero otherwise. Now $A_{0.5}$ is represented as follows:

$A_{0.5} = \{x, \mu_{A_{0.5}}(x)\}$
$=\{(0.1,0), (0.2,0), (0.3,1), (0.4,1), (0.5,1), (0.6,1), (0.7, 1), (0.8,0), (0.9,0), (1,0)\}$.

The similarity measure between A and A_{near} is calculated using Eq. (1), and between A and A^C by using Eq. (2). Various values of A_{near} are applied to compute similarity measures. The computation results are presented in Table 1.

Table 1. Similarity value between fuzzy set A and A_{near}, A and A^C.

Similarity measure	Measure value	Similarity measure	Measure value
$s(A, A_{0.1})$	0.64	$s(A, A^C)$	0.4
$s(A, A_{0.3})$	0.76		
$s(A, A_{0.5})$	0.80		
$s(A, A_{0.8})$	0.72		
$s(A, A_{0.95})$	0.56		

The dissimilarity measure between A and A_{near} is calculated using the result of Proposition 2.1, and between A and A^C by the result of Proposition 2.2. The computation results are also presented in Table 2.

Table 2. Dissimilarity value between fuzzy set A and A_{near}, A and A^C.

Dissimilarity measure	Measure value	Dissimilarity measure	Measure value
$D(A, A_{0.1})$	0.36	$D(A, A^C)$	0.6
$D(A, A_{0.3})$	0.24		
$D(A, A_{0.5})$	0.20		
$D(A, A_{0.8})$	0.28		
$D(A, A_{0.95})$	0.44		

The similarity measure for $s(A, A_{0.5})$ is calculated by using (1):

$$s(A, A_{0.5}) = 1/10(0.7+0.9+1+0.9+0.7) \\ +1/10(0.8+0.6+0.6+0.8+1) = 0.8.$$

Dissimilarity measure $D(A, A_{0.5})$ is also calculated by Proposition 2.1:

$$D(A, A_{0.5}) = 1/10(1+1+0.3+0.1+0.1+0.3+1+1+1). \\ +1/10(0.2+0.4+1+1+1+1+1+0.4+0.2) - 1 = 0.2$$

The remaining similarity measures and fuzzy entropies are calculated in a similar manner.

5. Conclusions

For different groups of data, there is a correlation between the degree of similarity and dissimilarity. Groups with a high degree of dissimilarity do not share enough common information. Hence, by analyzing the relationship between data, we verified that the totality of information contained by fuzzy sets constitutes the summation of the similarity measure and dissimilarity measure. First, we analyzed the similarity measure by means of a graphical comparison between a fuzzy set and comparative sets. With the results obtained, we deduced that the summation of similarity and dissimilarity between a fuzzy set and comparative set constitutes the totality of information of the fuzzy set. Results further indicate that the similarity and dissimilarity measures between fuzzy membership functions are complementary values. In the process of designing similarity and dissimilarity measures, we also proved the usefulness of the proposed measures. We also verified and discussed the similarity derivation with dissimilarity and dissimilarity derivation from similarity measure. In a simple example, the summation of similarity and dissimilarity measure computation results demonstrates their complementarity.

References

[1] Y. Rébillé, “Decision making over necessity measures through the Choquet integral criterion,” *Fuzzy Sets and Systems*, vol.157, no.23, pp.3025-3039, 2006.

[2] V. Sugumaran, G.R. Sabareesh and K.I. Ramachandran, “Fault diagnostics of roller bearing using kernel based neighborhood score multi-class support vector machine,” *Expert Systems with Applications*, vol. 34, no. 4, pp. 3090 ~ 3098, 2008.

[3] W. S. Kang and J. Y. Choi, “Domain density description for multiclass pattern classification with reduced computational load,” *Pattern Recognition*, vol.41, no.6, pp.1997-2009, 2008.

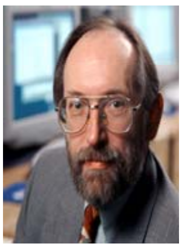
[4] F. Y. Shih and K. Zhang, “A distance-based separator representation for pattern classification,” *Image and Vision Computing*, vol.26, no.5, pp.667-672, 2008.

[5] S. M. Chen, "New methods for subjective mental workload assessment and fuzzy risk analysis," *Cybern. Syst. : Int. J.*, vol.27, no.5, pp.449-472, 1996.

[6] C. H. Hsieh and S. H. Chen, "Similarity of generalized fuzzy numbers with graded mean integration representation," *Proc. 8th Int. Fuzzy*

Systems Association World Congr., vol.2, pp.551-555, 1999.
[7] H. S. Lee, "An optimal aggregation method for fuzzy opinions of group decision," *Proc. 1999 IEEE Int. Conf. Systems, Man, Cybernetics*, vol.3, pp.314-319, 1999.
[8] S. J. Chen and S. M. Chen, "Fuzzy risk analysis based on similarity measures of generalized fuzzy numbers," *IEEE Trans. on Fuzzy Systems*, vol.11, no.1, pp.45-56, 2003.
[9] S. H. Lee, Y. T. Kim, S. P. Cheon and S. S. Kim, "Reliable data selection with fuzzy entropy," *LNAI*, vol.3613, pp.203-212, 2005.
[10] X. Liu, "Entropy, distance measure and similarity measure of fuzzy sets and their relations," *Fuzzy Sets and Systems*, vol.52, pp.305-318, 1992.
[11] S. K. Lin, "Gibbs Paradox and the Concepts of Information, Symmetry, Similarity and Their Relationship," *Entropy*, vol.10, pp.1-5, 2008.
[12] S. H. Lee, J. M. Kim, and Y. K. Choi, "Similarity measure construction using fuzzy entropy and distance measure," *LNAI*, vol.4114, pp.952-958, 2006.

Sang H. Lee received the B.S. degree in electrical engineering from Chungbuk National University, Cheongju, Korea, in 1988, the M.S and Ph.D. degrees in electrical engineering from Seoul National University, Seoul, Korea, in 1991 and 1998, respectively. He also received M.S. degrees in mathematics from Chungnam Naational University, Daejeon, Korea, in 2003. He served as a Research Fellow from 1996 to 1999 in the HOW Co. Ltd. Since 2000, he has been with Pusan National University. Currently he is a Chaired Professor in Specialized Group in Industrial Automation, Information, and Communication. His research interests include robust control theory, game theory, and fuzzy theory.



Witold Pedrycz is a Professor and Canada Research Chair (CRC - Computational Intelligence) in the Department of Electrical and Computer Engineering, University of Alberta, Edmonton, Canada. He main research directions involve Computational Intelligence, fuzzy modeling, knowledge discovery and data mining, fuzzy control including fuzzy controllers, pattern recognition, knowledge-based neural networks, relational computation, and Software Engineering. He has published numerous papers in this area. He is also an author of 13 research monographs covering various aspects of Computational Intelligence and Software Engineering. Witold Pedrycz has been a member of numerous program committees of IEEE conferences in the area of fuzzy sets and neurocomputing.

Dr. Pedrycz is intensively involved in editorial activities. He is an Editor-in-Chief of Information Sciences and Editor-in-Chief of IEEE Transactions on Systems, Man, and Cybernetics - part A. He currently serves as an Associate Editor of IEEE Transactions on Fuzzy Systems, and a number of other international journals. He has edited a number of volumes; the most recent entitled "Handbook of Granular Computing" (J. Wiley, 2008). In 2007 he received a prestigious Norbert Wiener award from the IEEE Systems, Man, and Cybernetics Council. He is a recipient of the IEEE Canada Computer Engineering Medal 2008.

Gyoyong Sohn received the B.S. degree in mathematics and computer science, in 2000, from Youngnam University, Korea, and the M.S. and Ph. D. degrees in mathematics from Kyungbuk National University, Deagu, Korea in 2002, and 2008 respectively. He has been a Post-doctoral course in College of electrical and computer engineering at Chungbuk National University since 2008. His research interests include the algebraic curve theory, information security, database security, and fuzzy theory.