Equality and Identity and (In)distinguishability in Classical and Quantum Mechanics from the Point of View of Newton's Notion of State[1]

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Abstract

The notion of state is a central notion for all branches of physics. Surprisingly enough, Newton's notion differs from the nowadays notion. Our review of the benefits of Newton's notion comprises Gibbs's paradox, Einstein's derivation of the classical and quantum distribution laws from the energetic spectrum (serving to remove anthropomorphic elements), the difference between 'identical' and 'indistinguishable' (being a property of states rather than of particles), a new physical content of $|\psi(x,t)|$ (the invariance of $|\psi(x,t)|$ rather than $\psi(x,t)$ against permutations yields not only fermions and bosons, but also anyons), and a novel classification of forces (leading eventually to a derivation of the Maxwell-Lorentz equations from classical mechanics).

1 Introduction

Classical mechanics is the safest (if not the *only* safe) ground we can move on. For this, we will analyze the implications of Newton's notion of state differing considerably from the contemporary one for the notions 'equality', 'identity' and '(in)distinguishability' playing a paramount role in statistics and in quantum mechanics. Newton's notion allows for considering them within classical point mechanics, what frees the discussion from anthropomorphic elements. Bach's (1997) fundamental results are obtained within an *elementary dynamical* framework.

2 Newton's notion of state

Newton's First Law "Every body perseveres in its state of being at rest or of moving uniformly straight forward, except insofar as it is compelled to change its state by forced impressed." –
Newton's state corresponds to nowadays' stationary state.

Newton's Second Law "A change in motion is proportional to the motive force impressed and takes place along the straight line in which that force is impressed." –
$\Delta\vec{p} \sim \vec{F}$: Newton's state variable is the momentum (the conserved quantities).

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Laplace's demon "A sufficiently powerful intelligence knowing all loci and velocities in a mechanical systems at one time is able to calculate the loci and velocities at all later times." –
Laplace's state is nowadays' state, including both stationary and non-stationary ones, the state variables are a complete set of independent dynamical variables.

Advantages of Newton's notion:

- Easily generalized to classical and quantum systems (not tied to orbits);
- Amount of conserved quantities ~ amount of quantum (state) numbers;
- Symmetry of state = symmetry of state function → gauge symmetry, geometric phases, . . .

Advantages of modern (Laplace's) notion:

- Complete description of motion;
- Identification of phase space points differing only by interchanging equal bodies → multiply connected spaces → appropriate topology for anyons.

Disadvantages of modern (Laplace's) notion:

- The state changes even in absence of causes;
- State at rest is ignored;
- Interchange of equal bodies changes state → Gibbs's paradox;
- Inapplicable to quantum systems.

3 Equal bodies in Newtonian states

1. Equal / identical bodies / particles

Equality We call two classical bodies or quantum particles equal, when their interchange does not change the properties / state / motion of a system. (*cf* Helmholtz, §10)

Identity "Particles are called identical, if they agree in all their intrinsic (i.e. state independent) properties." (Bach, p.15)
Remark: The restriction to the *intrinsic* properties circumvents the conflict with the logical notion 'identical' (= equal in *all* properties).

2. Permutation symmetry of Newtonian states

All conserved quantities of a classical-mechanical system: total energy / momentum / angular momentum / . . . , are invariant w.r.t. the permutation of equal parameters, *ie*, w.r.t. the permutation of (labels of) equal bodies, *ie*, w.r.t. bodies with equal properties concerning the system considered, ⇒

- A Newtonian state is invariant against interchanging (labels of) equal bodies,
- Equal bodies cannot be distinguished or identified by means of the conserved quantities (Newtonian/ stationary-state variables),
- Anthropomorphic arguments like 'particle can be marked or not' are not relevant points of view (interchanging or marking two resting red balls in a snooker game does not interfere the game);
- Indistinguishable classical particles have no trajectories (Bach provides probabilistic proof).

3. Comparison with Laplace's notion of state (continued)

The locus of a body is that part of space it occupies. Euler's exclusion principle (not to be interchanged with Pauli's exclusion principle!) states, that no body can occupy more than one locus, and no part of space can be occupied by more than one body. $\Rightarrow$

- Equal bodies can, at least in principle, always be distinguished and identified by means of their locus.
- Equal bodies are distinguishable within Laplace's notion of state.
- *(In)distinguishability is a property of states, not of particles/bodies.* (*cf* Bach, p.15)

4 Classical and quantum distribution laws

Einstein (1907) has derived the classical and quantum distribution laws using just the energetic spectra of a classical (continuous spectrum) and a quantum harmonic oscillator (discrete spectrum):

$$\langle E \rangle_{class} = \frac{\int_0^\infty E e^{-\frac{E}{kT}} dE}{\int_0^\infty e^{-\frac{E}{kT}} dE} = kT \tag{1}$$

$$\langle E \rangle_{quant} = \frac{\sum_{n=0}^\infty \hbar\omega n e^{-\frac{\hbar\omega n}{kT}}}{\sum_{n=0}^\infty e^{-\frac{\hbar\omega n}{kT}}} = \frac{\hbar\omega}{e^{\frac{\hbar\omega}{kT}} - 1} = \langle E \rangle_{class} \frac{\frac{\hbar\omega}{kT}}{e^{\frac{\hbar\omega}{kT}} - 1} \tag{2}$$

(In)distinguishability does not play any role (*cf* Bach's "Bose-Einstein statistics as invented by Boltzmann").

5 New physical content of $|\psi(x,t)|$

FFP5: Quantum-mechanical systems are conservative systems which may assume configurations for which $V(x) > E$

$\Rightarrow$ There is an *effective* potential energy

$$V_{nkl}(x) = V_{E_{nkl}}(x) = F_{E_{nkl}}(x) \cdot V(x) \leq E_{nkl}; \quad -\infty < x < +\infty \tag{3}$$

$F_{E_{nkl}}(x) \sim \left|\psi_{E_{nkl}}(x)\right|^2$ is a limiting function such, that $V_{nkl}(x) \leq E_{nkl}$ even if $V(x) > E_{nkl}$ ($\rightarrow$ tunnel effect demystified).

Progress since FFP5:

- Common principles of state change for classical and quantum systems $\rightarrow$ Eulerean derivation of time-dependent Schrödinger equation from time-independent one;
- $F_{E_{nkl}}$ is dimensionless $\Rightarrow$ $F_{E_{nkl}} = F_{E_{nkl}}(x/x_0)$: *All* quantum systems exhibit characteristic length x_0;
- $F_{E_{(nkl)}}(x/x_0) \sim |\psi_E(x)|^2 \rightarrow$ gauge symmetry;
- Novel classification of fields:
  A) Fields being related to total energy, E (accelerating fields like electric field, $\vec{E}$),
  B) Fields being related not to E, but to x_0 (refracting fields like magnetic field, $\vec{B}$),
  C) Fields being related neither to E, nor to x_0 (gauge fields, see Aharonov-Bohm effect);
  Remark: The *Maxwell-Lorentz equations* turn out to be just *compatibility conditions* for such fields $\vec{E}$ and $\vec{B}$.

6 Newtonian state variable $|\psi(x,t)|$

1) By virtue of their definition, limiting functions are invariant against permutations of equal particles (omitting x_0):

$$F_E(x_2, x_1) = F_E(x_1, x_2) \geq 0 \tag{4}$$

$\Rightarrow$ the most general representation of $F_E(x_1, x_2)$ reads (m_E entire)

$$F_E(x_1, x_2) = |\psi_E^{\gamma,m}(x_1, x_2)|^2 ; \tag{5}$$

$$\psi_E^{\gamma,m}(x_1, x_2) = \frac{1}{\sqrt{2}} e^{i\gamma_E(x_1,x_2)} \left[\tilde{\psi}_E(x_1, x_2) + e^{im_E\pi}\tilde{\psi}_E(x_2, x_1)\right] \tag{6}$$

2) Wigner's theorem:
If $\psi_E^{\gamma,m}(x_1, x_2)$ is eigenfunction, then

$$\hat{R}\psi_E^{\gamma,m}(x_1, x_2) = e^{i\rho_E^{(1,2)}} \psi_E^{\gamma,m}(x_2, x_1) \tag{7}$$

$$= e^{i\rho_E^{(1,2)}} e^{im_E\pi} e^{i[\gamma_E(x_2,x_1) - \gamma_E(x_1,x_2)]} \psi_E^{\gamma,m}(x_1, x_2) \tag{8}$$

is also eigenfunction $\Rightarrow$ $[\gamma_E(x_2, x_1) - \gamma_E(x_1, x_2)]$ can be absorbed into $\rho_E^{(1,2)}$.

3) 2^{nd} permutation:

$$\hat{R}^2\psi_E^m(x_1,x_2) = \hat{R}e^{i\rho_E^{(1,2)}}\psi_E^{\gamma,m}(x_2,x_1) = e^{i\rho_E^{(1,2)}}e^{i\rho_E^{(2,1)}}\psi_E^m(x_1,x_2) \quad (9)$$

Standard case:

$$\hat{R}^2 = e^{i\rho_E^{(1,2)}}e^{i\rho_E^{(2,1)}} = 1; \quad \rho_E^{(2,1)} = -\rho_E^{(1,2)}; \quad \gamma_E(x_1,x_2) = 0 \quad (10)$$

$\Rightarrow$ Wave functions are either symmetric (bosons, $+$) or anti-symmetric (fermions, $-$):

$$\psi_E^{\pm}(x_1,x_2) = \frac{1}{\sqrt{2}}\left[\tilde{\psi}_E(x_1,x_2) \pm \tilde{\psi}_E(x_2,x_1)\right] \quad (11)$$

Non-standard, 'anyonic' case:

$$\hat{R}^2 = e^{i\rho_E^{(1,2)}}e^{i\rho_E^{(2,1)}} \neq 1; \quad \rho_E^{(2,1)} \neq -\rho_E^{(1,2)}; \quad \gamma_E(x_2,x_1) - \gamma_E(x_1,x_2) \neq 0 \quad (12)$$

$\Rightarrow$ If there are topologically inequivalent paths, the wave function is neither symmetric, nor anti-symmetric, but can exhibit any intermediate behaviour $\rightarrow$ anyons (the clue to the fractional quantum Hall effect).

7 About the meaning of 'identical' and 'indistinguishable'

1) Some rigorous definitions

Equal means 'equal in some well-defined properties such as mass, density, shape, charge ..., but not in all'
Example: the 2 electrons in the ground state of He (they differ in s_z)
'equal' depends on the view, *ie*, which properties one is looking at.

Congruent means 'equal in all essential (geometric) properties, but not in locus'
Example: 2 red snooker balls of high quality

Identical means 'one and the same', *ie*, equal in all properties (strictly speaking, *no* exception at all)
Example: 2 squares of equal side length on the same place of a sheet

Indistinguishable means, that there is *no* mean (no one differing property/attribute) for discrimination.

$\Rightarrow$ Indistinguishable things are identical (Leibniz)

2) Questions

If we weaken the definition of 'identical', there may be a weakening of 'distinguishable' to 'identifiable', so that we are led to the question

- Are there non-classical indistinguishable bodies/particles not being identical?

- Are there principally distinguishable (*ie*, not identical) bodies/particles not being identifiable?

3) Observations

- Quantum particles are identical w.r.t. intrinsic properties and 'almost identical' w.r.t. state properties: All electrons (protons, ...) exhibit the same mass at rest, electrical charge, modulus of spin, etc.;
- Pauli's exclusion principle: 2 electrons differ in at least one quantum number – however: it does *not* say, which electron is in which state (entanglement);
- The quanta occupying an oscillator loose their individuality: Say, 12 quanta in state E_{12} occupy all together the *one* 12-quanta state, not 12 single-quantum states, $\Rightarrow$ they have got no individual properties (parameter values)
(in contrast to electrons, these quanta – Planck's "energy elements" – occupy not single-particle, but *single-system* states); – nevertheless: these 12 quanta are not one and the same (one thing) as we are thinking them as 12 particles.
- Cluster law: Wave functions of distinct systems need not to be entangled $\leftrightarrow$ There are distinguishable equal quantum particles.

4) Conclusions

- The notions 'identical' or 'distinguishable' as used in logics play almost no role, in fact: the actual physical meaning of 'distinguishability' as property of states is the *identifiability*;
- 'Identical' is meaningful in the sense of Bach (including only intrinsic properties, *eg*, spin s – but not s_z);
- There is no principal difference between classical bodies, bosons and fermions w.r.t. these properties.

8 Summary

Newton's notion of state is an addition to, though not a complete replacement of Laplace's notion. Our treatment of equality, identity and (in)distinguishability accounting for Newton's notion of state reveals the following advantages and new results.

- Common treatment of classical bodies and quantum particles;
- Non-probabilistic classification of (bodies/particles in) states;

- The limiting function F_E gives $|\psi_E|$ a new physical meaning as relative space occupation; both exhibit the same symmetry as Newtonian (stationary-) state variables (quantum numbers) w.r.t. external fields ($\rightarrow$ gauge invariance) and permutations ($\rightarrow$ fermions, bosons, *and anyons*);
- Distribution functions can be related to energetic spectra and occupation $\Rightarrow$ they are independent of (in)distinguishability;
- The meaning of 'identity' and 'indistinguishability' in physics is partly at variance with their meaning in logics, hence, careful restrictions are necessary;
- There are particles exhibiting different extrinsic properties (*eg*, spin direction in EPR), but not being identifiable: Equal bodies in symmetric states cannot be identified, be there different attributes or not.

References

[1] A. Bach, *Indistinguishable Classical Particles*, Berlin · Heidelberg: Springer, 1997 (Lecture Notes in Physics m44)

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[3] H. v. Helmholtz, *Introduction to the Lectures on Theoretical Physics*, Leipzig: Barth, 1903 (in German)